Wavimeter

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Abstract— **A method of extracting wave parameters from wave height measured from a moving platform is presented. A least squares approach is presented to estimate wave parameters using vessel velocity and perceived wave frequency that eliminates Doppler due to vessel motion. Two techniques for estimating wave frequency are presented: a block data method using MUSIC and a real-time method using demodulation. Sea trial results demonstrate that this method is as effective as a WaveRider buoy for estimating wave parameters.**

I. Introduction

This paper presents the wavimeter[1], a system for the measurement of ocean wave parameters from a moving vessel using an acoustic ranger and an inertial heave sensor. The wavimeter produces a heave corrected time series of instantaneous wave height relative to mean sea level. From this time series are found estimates of the wavelength and direction of the principle wave. Results of sea trials are given below in which the estimated wave parameters agree very closely with those measured using a Waverider buoy.

The wavimeter uses the same sensors as the depthimeter. The design and operation of the depthimeter and the measurement of wave height from a moving platform is described completely in [2], [3]. Just as the depthimeter combines data from an acoustic ranger and a heave sensor to estimate vessel depth, the filtered range from an acoustic ranger can be added to the heave to estimate wave height. Distance to the water surface is found using an uplooking acoustic ranger, the Tritech ST500-6 with a range of 0.3 to 50 m and a resolution of 1.2 cm.

This system was developed as part of an ongoing effort to accurately measure depth and to correct UUV bathymetry for heave. The resulting bathymetric data have met IHO standards for a special hydrographic survey (International Hydrographic Organization 1998[4]) which requires a constant depth error of no greater than 0.25 m.

In the following section, we present a simple wave model, followed by the derivation of a estimator of principle wave parameters. Results of sea trials are presented comparing the parameters estimated by the wavimeter and by a WaveRider buoy.

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II. Wave model

Let $w(t, \boldsymbol{\alpha})$ represent water surface displacement from local sea level at time t and position $\boldsymbol{\alpha} = (x, y)$. If we assume that w can be modeled as the sum of partial waves, then

$$w(t, \boldsymbol{\alpha}) = \sum_{n=0}^{\infty} a_n \cos(\omega_n t - \mathbf{k}_n^T \boldsymbol{\alpha} + \phi_n) \tag{1}$$

where
a_n is the amplitude of the n-th partial wave,
ω_n is the angular frequency of the n-th partial wave,
$\mathbf{k}_n$ is the wavenumber of the n-th partial wave, a vector with x and y components, and
ϕ_n is the phase of the n-th partial wave.
This can also be written as a complex series

$$w(t, \boldsymbol{\alpha}) = \sum_{n=-\infty}^{\infty} c_n e^{i(\omega_n t - \mathbf{k}_n^T \boldsymbol{\alpha})} \tag{2}$$

where the initial phase ϕ_n has been absorbed into the complex coefficients, c_n. For equivalence with the real series in (1), it follows that $\omega_n = -\omega_{-n}$ and $\mathbf{k}_n = -\mathbf{k}_{-n}$ with the result that the coefficients are hermitian symmetric, $c_n = c_{-n}^*$.

This basic model can be specialized for the case of a moving platform. Assuming the time varying position is known and that all other parameters are time invariant, the surface displacement can be written as a function of time only:

$$w(t) = \sum_{n=-\infty}^{\infty} c_n e^{i(\omega_n t - \mathbf{k}_n^T \boldsymbol{\alpha}(t))} \tag{3}$$

This corresponds to the observed wave height from the moving platform, and it includes a Doppler shift in frequency of each partial wave. The observed frequency of the n-th partial wave, $\theta_n(t)$ and the true angular frequency are related by

$$\theta_n(t) = \frac{d}{dt}(\omega_n t - \mathbf{k}_n^T \boldsymbol{\alpha}(t)) = \omega_n - \mathbf{k}_n^T \mathbf{v}(t)$$

where $\mathbf{v}(t)$ is the velocity of the platform. Note that different true frequencies can result in the same observed frequency. For a single partial wave, this shift reaches a critical value at $\omega_n = \mathbf{k}_n^T \mathbf{v}$, at which point the platform is moving at exactly the same velocity as the partial wave in the direction normal to the wave front.

III. Estimation of wave parameters

Complete recovery of the parameters c_n, ω_n, and $\mathbf{k}_p$ from a set of observed wave spectra is difficult due to the ambiguity that the Doppler shift introduces. However, it is possible to extract an estimate of the frequency of the principal wave by finding the peak in the observed wave spectrum.

In this paper, it is assumed that there is a single significant peak in the spectrum of the observed wave heights at ω_p corresponding to a principal partial wave. For this case the peak observed frequency is modeled as

$$\theta_p(t) = \omega_p - \mathbf{k}_p^T \mathbf{v}(t) \tag{4}$$

Using (4), a general linear estimator is derived for the unknown wave parameters ω_p and $\mathbf{k}_p$ solved using standard least squares methods. Two approaches are presented for estimating measured altimeter spectra, one for constant velocity survey lines, and one for arbitrary course and velocity.

A. The general linear estimator

When the velocity of the moving platform is known, it is possible to produce a linear estimator for ω_p and $\mathbf{k}_p$ if the frequency of the spectral peak in the observed wave height can be accurately determined. Given an observed set of N peak frequencies and velocities, $\{\theta_n, \mathbf{v}_n\}_{n=1}^N$, (4) can be written as

$$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_N \end{bmatrix} = \begin{bmatrix} 1 & -\mathbf{v}_1^T \\ 1 & -\mathbf{v}_2^T \\ \vdots & \vdots \\ 1 & -\mathbf{v}_N^T \end{bmatrix} \begin{bmatrix} \omega_p \\ \mathbf{k}_p \end{bmatrix} + \mathbf{e} \tag{5}$$

where $\mathbf{e}$ is the modeling error. This can be solved for $\hat{\omega}_p$ and $\hat{\mathbf{k}}_p$ using standard least squares methods. It follows that a minimum of three tracts run in different directions are necessary for a unique solution.

In practice, determining the spectral peak as a function of time is the most difficult part of the estimation problem.

B. Estimation of the spectral peak

Estimation of wave parameters requires that the peak in the spectrum of the observed wave height, θ_p, be determined accurately. Since the time series involved are not generally well behaved, many traditional methods lack the noise immunity and precision necessary for this application. Two approaches were found that consistently produced accurate results: Root MUSIC and demodulation. Each is best suited to particular survey types, although demodulation was found to perform well under most circumstances.

The advantage of straight-line, constant velocity survey lines is the longer time window available over which stationarity of measured altitude data can be assumed. This allows greater flexibility in estimating the spectrum of the surface waves. In this case Root MUSIC and demodulation achieve comparable results.

For survey lines with arbitrary velocity $\boldsymbol{\alpha}(t)$, a similar approach can be taken by expanding $\boldsymbol{\alpha}(t)$ in a Taylor series over a short period and retaining only the first and second terms. This has the effect of linearizing motion over short distances. Unfortunately, this results in a relatively short time window, significantly decreasing the reliability of spectral estimates which assume stationarity. An alternative approach is to use the true path. As a result the spectrum must be assumed to be non-stationary, and demodulation proved the best approach.

1) MUSIC

Root MUSIC is a eigen-based, high resolution estimator for spectral peaks. It employs the eigenvalues and eigenvectors of the sample covariance matrix to produce an estimate of the the spectrum from which the peak is easily extracted. An excellent treatment of this method is given in [5].

MUSIC returns a single spectrum for a given time series, thus it is applicable in the special case of a straight line survey with constant velocity.

2) Demodulation

Demodulation is a process of extracting instantaneous frequency from a time series. Its use requires that $w(t)$ be a relatively narrowband signal with a strong spectral peak. An advantage of demodulation is that it can be used under any survey conditions as it returns a time varying estimate of ω_p. Demodulation consists of the following steps:

1. Take the Hilbert transform of $w(t)$ by filtering or with the FFT:

$$\breve{w}(t) = \mathcal{H}(w(t))$$

2. Make an analytic signal from $w(t)$:

$$w_A(t) = w(t) + i\breve{w}(t)$$

3. Extract the phase of the analytic signal:

$$\phi(t) = \arg(w_A(t))$$

4. Unwrap the phase (remove jumps of 2π in $\phi(t)$). Phase unwrapping is essential at this point as the next step is differentiation. In practice, this proved to be easily accomplished by looking for jumps in phase near 2π and adding multiples of 2π to the remainder of the waveform.
5. Differentiate the unwrapped phase and average or low pass filter to extract frequency:

$$\hat{\theta}_p = \text{avg}(\frac{d}{dt}\phi(t));$$

A simple average or a low pass filter can be used at this stage. The average is used for straight line surveys with constant velocity where a single frequency is expected. A low pass filter with zero phase shift is used for survey lines that are not straight or have time varying velocity.

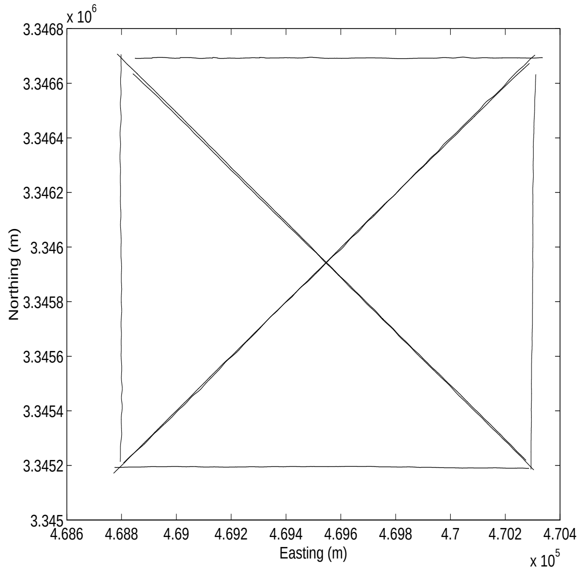

Fig. 1. Straight-line survey tracks.

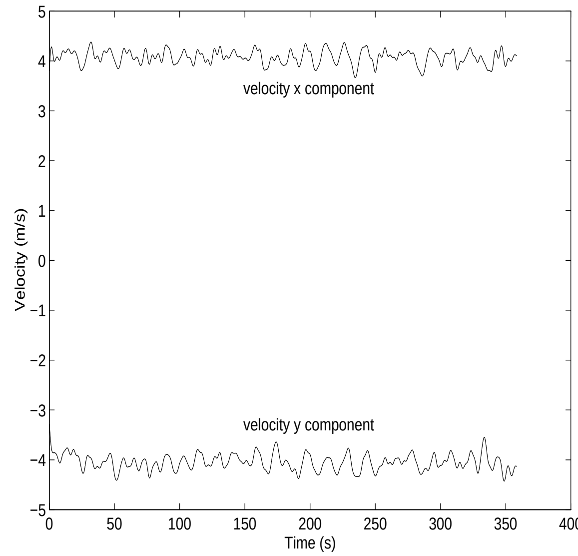

Fig. 2. Velocity x and y components versus time for one track in straight line survey.

IV. Results

A. Description of data

Data was collected with the ORCA[6] on 15 August 1998. Wave height and x,y position were sampled at 10 samples per second. Wave height was collected using a Tritech model ST500 altimeter operating at 500 kHz with a 6 degree beam. The altimeter was configured as an up-looking sonar and when corrected for vessel heave was accurate to within 5 cm. Positioning and heave were measured using the Applied Analytics POS/MV 320 which combines GPS and an inertial system with a better than 5 m positioning accuracy. An independent measurement of wave parameters was taken by a Directional Waverider Mark II 375 near the center of the survey area. Two surveys were taken, a series of eight straight lines and a circle.

Some preprocessing of the data was performed. Missing values in the range data were found and replaced by linear interpolation of neighboring values. The heave data were resampled from 10 to 5 samples per second and synchronized with the range data.

1) The straight line survey

The straight line survey is shown in Fig. 1. It consists of eight tracks (the diagonals are taken in both directions) resulting in eight lines with headings at multiples of 45 degrees from 0 to 315 degrees. Each side of the surveyed area is 1500 meters. At a nominal speed of 10 knots each line took from 4 to 7 minutes. The entire survey took just under one hour 10 minutes.

Two time series are required to estimate wave parameters, the vehicle velocity and the wave height (x, y, z) triples. In Fig. 2 the x and y components of the measured velocity on one of the eight tracks (moving northeast to southwest) are plotted versus time. Normal variation is evident in the velocity due to an active control system and environmental factors. Applying the techniques outlined above, the wave height was computed for each track and the time series for the same track is shown in Fig. 3.

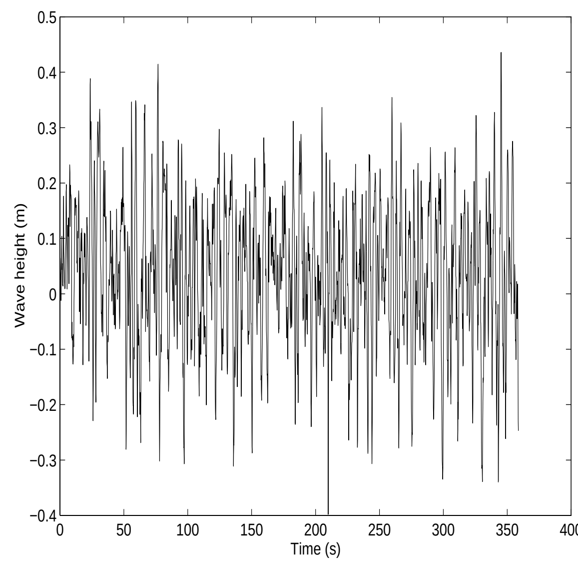

Fig. 3. Wave height versus time.

2) The circle survey

The circle survey was taken in the same area as the straight line survey, and consisted of a single circular path 1000 meters in diameter as shown in Fig. 4. At a nominal speed of 10 knots, the survey took under 10 minutes. It is interesting to note that the circle survey was considerably shorter in duration than the straight line survey, yet it yielded results of quality comparable with those of the straight line survey.

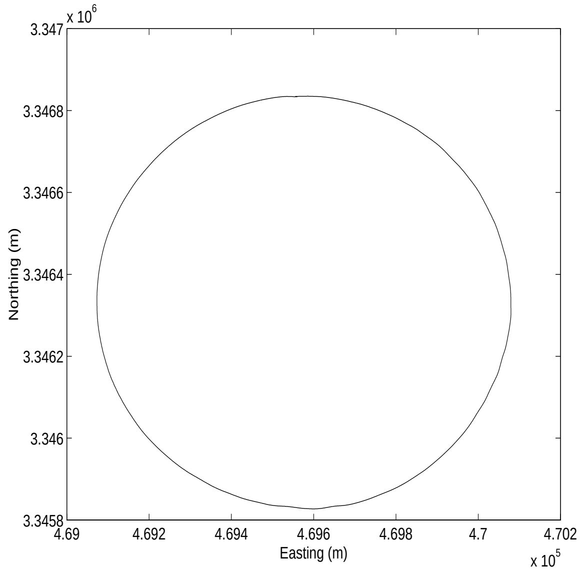

Fig. 4. Circle survey.

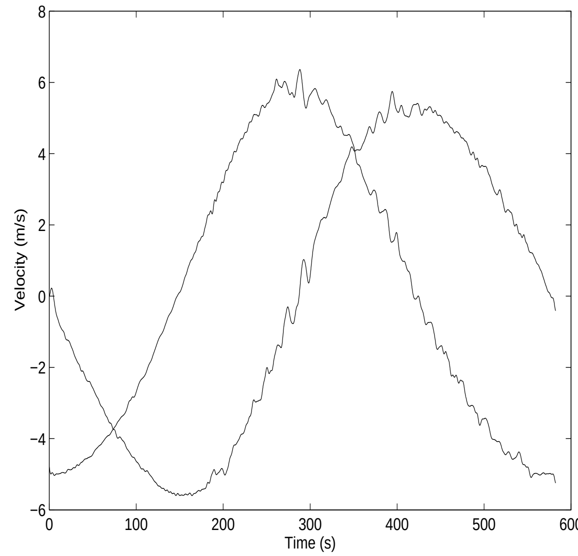

Fig. 6. Circle x and y velocity versus time.

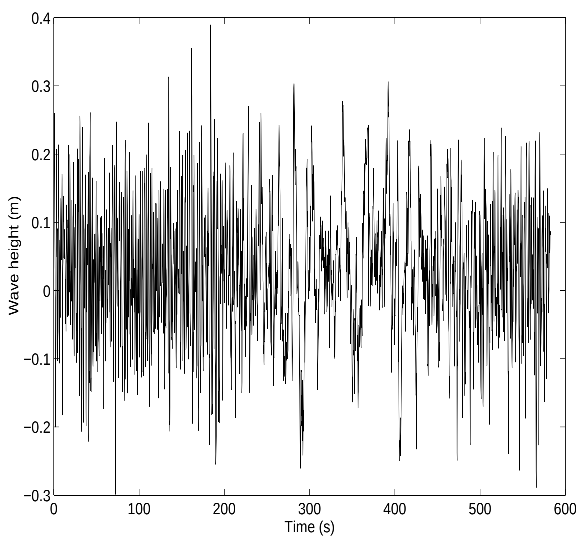

Fig. 5. Circle wave height versus time.

As with the straight line survey, two time series are required to estimate wave parameters, the vehicle velocity and the wave height (x,y,z) triples. These series are plotted versus time in Figs. 5 and 6 for the entire survey. Note that the frequency shift due to changing heading can be clearly seen in Fig. 5.

B. Estimation of wave parameters

The generalized linear estimator is employed for both of these surveys. As described above, this produces estimates of ω_p and $\mathbf{k}_p$ given the velocity of the platform and the spectral peak frequency in the wave height. These estimates are then compared to the values measured simultaneously using a WaveRider buoy.

The chief difficulty in this approach is the determination of peak observed frequency. Two factors play into this difficulty: relatively short duration records and non-stationarity of the random process. For the straight line survey, the spectral peak is estimated using both MUSIC and demodulation. Because the velocity is constantly changing for the circle survey, only the demodulation approach can be used.

1) The straight line survey

The first method used to analyze the straight line survey data is MUSIC. The average velocity is computed for each track, and the wave height time series is block processed using MUSIC to determine the spectral peak observed for each track. This results in eight velocities and frequencies that are used with the generalized linear estimator to produce the estimated wave parameters.

The observed peak frequencies are plotted in Fig. 7. On this polar plot, the distance from the origin is proportional to observed frequency and the angle corresponds to vessel heading. The peak frequency for each track as estimated using MUSIC is plotted as an $\times$. The arrow in Fig. 7 indicates the estimated wave direction and frequency, and the theoretical observed frequency/heading curve is plotted using the estimated wave parameters. When the principle wave component is constant during the measurement interval, the plot of observed wave frequency versus vessel heading and assuming constant vessel speed is a Pascal limaçon. Note that the minimum observed frequency corresponds to the vessel heading in the same direction as the wave front.

The parameter estimates are converted to the same form as those produced by the WaveRider and are given in Table I for comparison.

The second method uses demodulation to estimate the spectral peak as a time series of instantaneous values. In this case, the peak and and velocity time series are used in

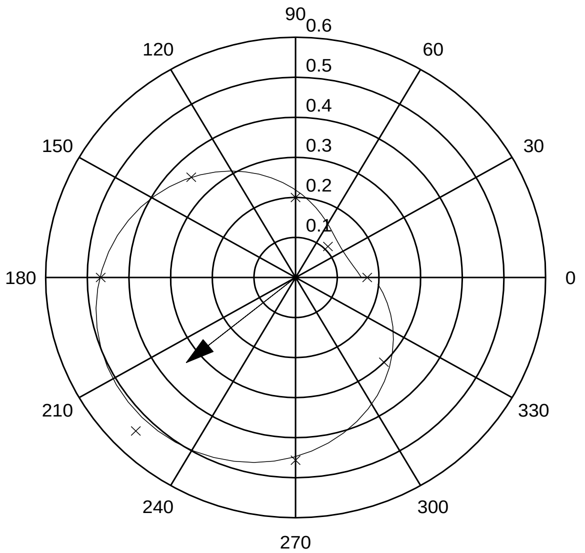

Fig. 7. Estimated and predicted wave frequency versus heading using MUSIC for straight line survey.

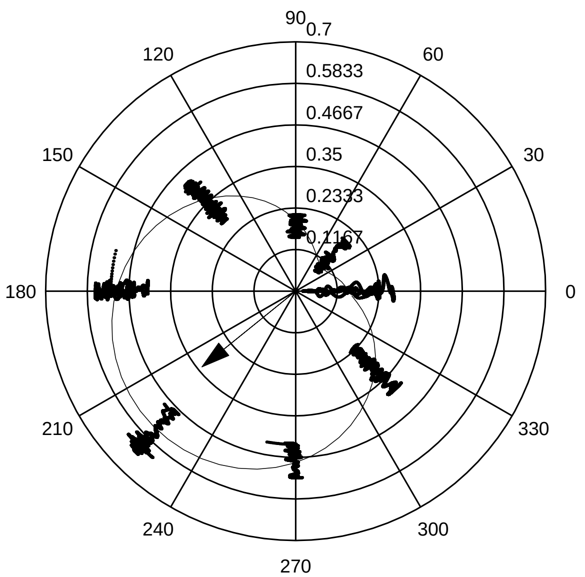

Fig. 8. Estimated and predicted wave frequency versus heading using demodulation for straight line survey.

the linear estimator to produce the parameter estimates. As with the previous method, the peak values are plotted on a polar plot versus vessel heading in Fig. 8. Rather than a single value for each track, the result is a cluster of values clustered on the theoretical peak observed frequency/heading limaçon. The variation in peak frequency is the result of changes in vessel velocity, nonstationarity of the waves and error in the estimator. The estimated wave direction and frequency are represented by the arrow, and the converted results are given in Table I.

For this survey, both MUSIC and demodulation produce results that are consistent with WaveRider estimates within one percent judge which is more effective.

TABLE I
Results of analysis of straight-line survey using root MUSIC and demodulation.

	WaveRider	Wavimeter	
		MUSIC	demod
f_p	0.338	0.332	0.340
T_p	2.96	3.01	2.94
ϕ	219	219.3	218.6
λ		25.8	23.5

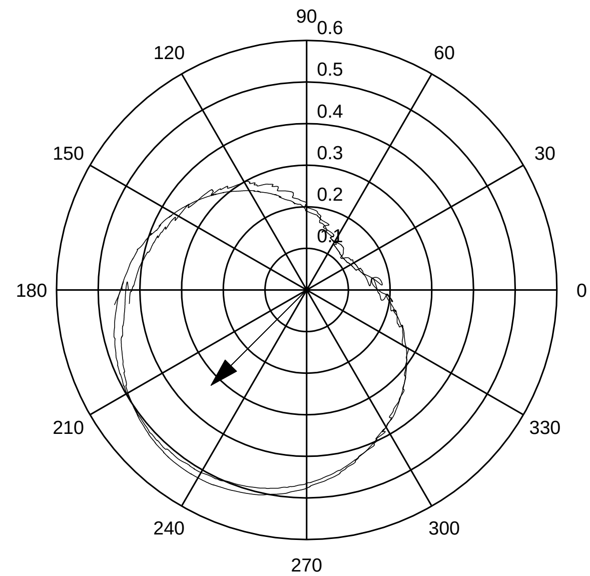

Fig. 9. Estimated and predicted wave frequency versus heading using demodulation for the circle survey.

2) The circle survey

Although it was performed at a nearly constant speed, the shape of the circle survey results in x and y components of velocity that change constantly with time. As a result, the MUSIC or other block processing techniques are not effective. In this case only the demodulation method is employed. This survey has a significant advantage over the straight line survey in that a diversity of headings is available for the linear estimator, and this results in excellent estimates with a much shorter data collection period.

In Fig. 9 the estimated peak frequency is plotted versus heading superimposed on the theoretical frequency/heading curve. For this data set, the curve is an approximate limaçon since the measured velocity is used to generate the curve. Because the survey includes a full circle, there are frequency estimates at all headings, and this time series along with the measured velocity is used in the linear estimator to estimate the wave parameters. The resulting estimated wave direction and frequency is shown as an arrow in Fig. 9.

The results are converted to WaveRider format and given in Table II. Comparing the results with those of the

TABLE II
RESULTS OF ANALYSIS OF CIRCLE SURVEY.

	WaveRider	Wavimeter demod
f_0	0.325	0.325
T_p	3.08	3.08
ϕ	225	226
λ		28.6

WaveRider demonstrates a remarkable consistency, especially give the relative short survey time of less than ten minutes.

V. Conclusions

The wavimeter provides results equivalent to those of the WaveRider buoy. Several approaches to this problem are presented in this paper. Although all proved effective in sea trials, there were distinct differences that became apparent in the trials.

Two survey types were employed in the sea trials, the circle and straight line surveys. The straight line survey works well with block data processing and is easier to navigate, lending itself well to typical sidescan or bathymetric surveys. The circle survey requires less data collection to achieve comparable results because of a greater diversity of headings in the data, but it requires a more complex real-time demodulator and more computation in the parameter estimator.

The two methods of wave frequency estimation used in this paper each proved accurate. The block data processing method using MUSIC is efficient and requires little storage, but its usefulness is limited to survey types with blocks of data collected at constant or nearly constant vessel velocity. Demodulation is fast and accurate, and it requires less data collection. However, it is considerably more complex. As implemented in this paper it requires a Hilbert transformer, a phase unwrapper and a differentiator. While this could be considerably simplified through the use of a line detector/tracker, the resulting system would still be more complex than the block processing method.

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